Math 53 (Multivariable Calculus), Section 102 & 108
Week 3, Friday
Sep 9, 2022
For the other materials: seewoo5.github.io/teaching/2022Fall

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1. Find the equation of a sphere centered at $(1, 1, 2)$ and passes through the origin. Find the points that the sphere meets with x, y, and z-axis.
2. Find the equation of set of all points equidistant from the points $A(1, 2, 3)$ and $B(5, 4, 3)$. Describe the set.
3. Find equation**s** of sphere**s** of radius 2 that touches xy, yz, and xz-planes.

Solution

1. The equation has a form of $(x-1)^2+(y-1)^2+(z-2)^2=r^2$, where r is the radius of the sphere. Since it passes through the origin, we have $(0-1)^2+(0-1)^2+(0-2)^2=r^2 \Rightarrow r=\sqrt{6}$. Hence the equation becomes

$$(x-1)^2+(y-1)^2+(z-2)^2=6 \Leftrightarrow x^2+y^2+z^2-2x-2y-4z=0.$$

x axis can be thought as a set of points where $y=z=0$. This gives $x^2-2x=x(x-2)=0 \Rightarrow x=0,2$, so the sphere meets with x-axis at the origin $(0,0,0)$ and $(2,0,0)$. Similarly, the sphere meets with y-axis at $(0,0,0)$ and$(0,2,0)$, and with z-axis at $(0,0,0)$ and $(0,0,4)$.

2. Let (x,y,z) be a point in the set. We have

$$\begin{aligned}
&\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}=\sqrt{(x-5)^2+(y-4)^2+(z-3)^2}\\
&\Leftrightarrow (x-1)^2+(y-2)^2+(z-3)^2=(x-5)^2+(y-4)^2+(z-3)^2\\
&\Leftrightarrow x^2+y^2+z^2-2x-4y-6z+14=x^2+y^2+z^2-10x-8y-6z+50\\
&\Leftrightarrow 8x+4y=36\\
&\Leftrightarrow 2x+y=9
\end{aligned}$$

It is a plane perpendicular to the xy-plane and intersects with xy-plane as a line defined by the equation $2x+y=9$.

3. A sphere with radius r centered at (a,b,c) touches xy-plane if the distance between (a,b,c) and xy plane equals to the radius, and this occurs when $c=\pm r$. Similarly, we have $a=\pm r$ and $b=\pm r$. Since the radius $r=2$, we get the following 8 possible equations of spheres:

$$(x\pm 2)^2+(y\pm 2)^2+(z\pm 2)^2=2^2$$

where we have total 8 possibilities for choosing the signs. We can simplify them as follows:

$$\begin{aligned}
x^2+y^2+z^2+4x+4y+4z+8=0\\
x^2+y^2+z^2+4x+4y-4z+8=0\\
x^2+y^2+z^2+4x-4y+4z+8=0\\
x^2+y^2+z^2+4x-4y-4z+8=0\\
x^2+y^2+z^2-4x+4y+4z+8=0\\
x^2+y^2+z^2-4x+4y-4z+8=0\\
x^2+y^2+z^2-4x-4y+4z+8=0\\
x^2+y^2+z^2-4x-4y-4z+8=0
\end{aligned}$$