Math 53 (Multivariable Calculus), Section 102 & 108
Week 4, Friday
Sep 16, 2022
For the other materials: seewoo5.github.io/teaching/2022Fall

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1. Find parametric equations for the line through the point $(0, 1, 2)$ that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t, y = 1 - t, z = 2t$.

2. Let $\mathbf{n} = \langle 1, 1, 2 \rangle$ be a vector.

   (a) Find an equation of plane that is orthogonal to $\mathbf{n}$ and passes through the origin.

   (b) Find an equation of plane that is orthogonal to $\mathbf{n}$ and passes through $(1, 1, 1)$.

   (c) Find a distance between these planes.

3. Find an equation for the surface consisting of all points that are equidistant from the point $(-1, 0, 0)$ and the plane $x = 1$. Identify the surface.

Solution

1. Let $\mathbf{v} = \langle a, b, c \rangle$ be a vector represents the direction of the line. Since it is parallel to the plance $x+y+z=2$, it is *orthogonal* to the normal vector of the plane, which is $\mathbf{a} = \langle 1, 1, 1 \rangle$. Also, $\mathbf{v}$ is orthogonal to the line $x = 1+t, y = 1-t, z = 2t$, whose directional vector is $\mathbf{b} = \langle 1, -1, 2 \rangle$. Hence $\mathbf{v}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$, so that we can find $\mathbf{v}$ using a cross product:
$$\mathbf{v} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = \langle 3, -1, -2 \rangle.$$
Since the line passes through $(0, 1, 2)$, its parametric equation becomes
$$x = 3t, y = 1-t, z = 2-2t.$$

2. (a) We have $\mathbf{n} \cdot \langle x, y, z \rangle = 0 \Leftrightarrow x + y + 2z = 0$.

   (b) We have $\mathbf{n} \cdot \langle x-1, y-1, z-1 \rangle = 0 \Leftrightarrow (x-1) + (y-1) + 2(z-1) = 0 \Leftrightarrow x + y + 2z - 4 = 0$.

   (c) To find a distance between two parallel planes, we can pick any point from a plane and compute the distance between the point and the other plane. Hence we choose a point $(0, 0, 0)$ from the first plane, and the distance between it and the second plane $x + y + 2z - 4 = 0$ is
$$\frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|1 \cdot 0 + 1 \cdot 0 + 2 \cdot 0 + 4|}{\sqrt{1^2 + 1^2 + 2^2}} = \frac{4}{\sqrt{6}}.$$

3. Let (x, y, z) be a point on the surface. Then the distance between the point and $(-1, 0, 0)$ is $\sqrt{(x+1)^2 + (y-0)^2 + (z-0)^2}$, and the distance to the plane $x = 1$ is $\sqrt{(x-1)^2 + 0^2 + 0^2}$. Hence
$$(x+1)^2 + y^2 + z^2 = (x-1)^2 \Leftrightarrow y^2 + z^2 = (x-1)^2 - (x+1)^2 = -4x \Leftrightarrow x = -\frac{y^2}{4} - \frac{z^2}{4}$$
and this is an elliptic paraboloid.