Math 53 (Multivariable Calculus), Section 102 & 108
Week 6, Friday
Sep 30, 2022
For the other materials: seewoo5.github.io/teaching/2022Fall

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1. Compute $f_x, f_y, f_{xx}, f_{xy}, f_{yx}, f_{yy}$ of $f(x, y) = \sin(xy)$. Check that $f_{xy} = f_{yx}$.

2. Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$.

   (a) $\sin x + \sin y + \sin z = 1$

   (b) $e^x + e^y + e^z = xyz$

3. If

$$f(x, y) = y \tan^2(x^2) + \frac{x + y}{(x^2 + y^2)^{3/2}} e^{\sin(x\sqrt{y})}$$

   compute $f_x(1, 0)$ and $f_y(0, 1)$.

Solution

1.

$$\begin{aligned} f_x(x,y) &= y\cos(xy) \\ f_y(x,y) &= x\cos(xy) \\ f_{xx}(x,y) &= -y^2\sin(xy) \\ f_{xy}(x,y) &= \cos(xy) - xy\sin(xy) \\ f_{yx}(x,y) &= \cos(xy) - xy\sin(xy) \\ f_{yy}(x,y) &= -x^2\sin(xy) \end{aligned}$$

2. (a) For $\partial z/\partial x$, we have

$$\cos x + \cos z \cdot \frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{\cos x}{\cos z}$$

and similarly we get $\frac{\partial z}{\partial y} = -\frac{\cos y}{\cos z}$.

(b) If we differentiate with respect to x, we have

$$e^x + e^z\frac{\partial z}{\partial x} = yz + xy\frac{\partial z}{\partial x} \Rightarrow \frac{\partial z}{\partial x} = \frac{yz - e^x}{e^z - xy}$$

and similarly we get $\frac{\partial z}{\partial y} = \frac{xz - e^y}{e^z - xy}$.

(c) You DON'T have to differentiate the complicated function at all. The $f_x(a,b)$ is, by definition, equals to

$$f_x(a,b) = \lim_{h\to 0}\frac{f(a+h,b) - f(a,b)}{h}$$

and this is the same as $g'(a)$, where $g(x) = f(x,b)$ (fix y as b and regard as a function in x). If we plug $y = 0$, one can find that most of the complicated terms disappear and simplifies as $f(x,0) = 1/x^2$. Hence

$$f_x(1,0) = \frac{d}{dx}\Big|_{x=1}\frac{1}{x^2} = -2$$

Similarly, $f(0,y) = 1/y^2$ and $f_y(0,1) = -2$.