1 Vectors, Coordinates, Dimension

1. What is $(2,4)+(9,-2)$?

1. ____________

2. What is $(-1,5,2)-(0,-2,5)$?

2. ____________

3. What is $(4,-5,7)+(-3,-4)$?

3. ____________

4. What is the zero vector in $\mathbb{R}^5$?

4. ____________

5. If $v=(5,5,-1)$, what is $-v$?

5. ____________

6. Does $(-6,-7)$ equal $(-7,-6)$?

6. ____________

7. What does the equation $x=3$ represents in $\mathbb{R}^2$? How about in $\mathbb{R}^3$?

7. ____________

8. Find the equation of a line in $\mathbb{R}^2$ that passes through $(-1,1)$ and $(1,3)$. What is the slope of the line? What is the y-intercept?

8. ____________

9. What is the equation of the circle centered at the origin and the radius 3?

9. ____________

10. What is the equation of the circle centered at $(2,-1)$ and the radius 4?

10. ____________

11. What is the equation of the sphere centered at $(2,-1,3)$ and the radius 1?

11. ____________

12. What is the center and the radius of the circle given by the equation $x^2 + (y+1)^2 = 16$?

12. ____________

13. What is the center and the radius of the circle given by the equation $x^2 - 4x + y^2 + 6y = 12$?

13. ____________

14. What is the center and the radius of the sphere given by the equation $x^2 + y^2 + z^2 = x + y + z$?

14. ____________

2 Inner Products

1. What is $(2, 4) \cdot (9, -2)$?

1. ____________

2. What is $(-1, 5) \cdot (0, -2)$?

2. ____________

3. What is $(-1, 5, 2) \cdot (0, -2, 5)$?

3. ____________

4. If you were to assign a number between 0 and π to the angle between $(-1, 5)$ and $(0, -2)$, would it be less than, equal to, or greater than $\pi/2$?

4. ____________

5. If you were to assign a number between 0 and π to the angle between $(-1, 5, 2)$ and $(0, -2, 5)$, would it be less than, equal to, or greater than $\pi/2$?

5. ____________

Solutions

Vectors, Coordinates, Dimension

1. You can add two vectors with coordinates by adding each coordinate - $(2,4)+(9,-2)=(11,2)$.
2. $(-1,5,2)-(0,-2,5)=(-1-0,5-(-2),2-5)=(-1,7,3)$.
3. Sum of vectors with different dimensions is not defined.
4. $(0,0,0,0,0)$.
5. $-(5,5,-1)=(-5,-5,-(-1))=(-5,-5,1)$.
6. No.
7. In $\mathbb{R}^2$, it is a line that is perpendicular to x-axis (parallel to y-axis) and passes through $(3,0)$. In $\mathbb{R}^3$, it is a plane that is parallel to yz-plane and passes through $(3,0,0)$.
8. The slope of the line is $(3-1)/(1-(-1))=1$, so the equation has a form of $y=x+b$ for a y-intercept b. Since it passes $(-1,1)$, we have $1=(-1)+b$ and $b=2$. So the equation of the line is $y=x+2$ and its y-intercept is 1.
9. $x^2+y^2=3$.
10. $(x-2)^2+(y+1)^2=4^2$.
11. $(x-2)^2+(y+1)^2+(x-3)^2=1$.
12. It has a form of $(x-0)^2+(y+1)^2=4^2$, so the center is $(0,-1)$ and the radius is 4.
13. Once you complete the squares, you will get

$$\begin{aligned}
&x^2-4x+y^2+6y=12\\
\Leftrightarrow\ &(x-2)^2-2^2+(y+3)^2-3^2=12\\
\Leftrightarrow\ &(x-2)^2+(y+3)^2=2^2+3^2+12=25=5^2.
\end{aligned}$$

   So the center is $(2,3)$ and the radius is 5.

14. Once you complete the squares, you will get

$$\begin{aligned}
&x^2+y^2+z^2=x+y+z\\
\Leftrightarrow\ &x^2-x+y^2-y+z^2-z=0\\
\Leftrightarrow\ &\left(x-\frac{1}{2}\right)^2-\frac{1}{4}+\left(y-\frac{1}{2}\right)^2-\frac{1}{4}+\left(z-\frac{1}{2}\right)^2-\frac{1}{4}=0\\
\Leftrightarrow\ &\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2+\left(z-\frac{1}{2}\right)^2=\frac{3}{4}=\left(\frac{\sqrt{3}}{2}\right)^2.
\end{aligned}$$

   So the center is $(1/2,1/2,1/2)$ and the radius is $\sqrt{3}/2$.

Inner products

1. $2 \times 9 + 4 \times (-2) = 10$

2. $(-1) \times 0 + 5 \times (-2) = -10$

3. $(-1) \times 0 + 5 \times (-2) + 2 \times 5 = 0$

4. The cosine of the angle θ is

$$\cos\theta = \frac{(-1,5)\cdot(0,-2)}{\|(-1,5)\|\|(0,-2)\|} = \frac{-10}{\sqrt{26}\times 2} < 0.$$

   Now, for $0 \le \theta \le \pi$, from the graph of cosine, we have

$$\cos\theta \begin{cases} > 0 & 0 \le \theta < \frac{\pi}{2} \\ = 0 & \theta = \frac{\pi}{2} \\ < 0 & \frac{\pi}{2} < \theta \le \pi \end{cases}$$

   so the angle is between $\pi/2$ and π. Actually, you can find that only the sign of the dot product is important, not the lengths of vectors.

5. Similarly, we have

$$\cos\theta = \frac{(-1,5,2)\cdot(0,-2,5)}{\|(-1,5,2)\|\|(0,-2,5)\|} = 0$$

   so the angle is exactly $\pi/2$.