1 Matrix Algebra

1. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$. Find $A + B$.

1. _____

2. Let $C = \begin{bmatrix} 3 & -5 & 2 \\ 1 & 0 & 4 \end{bmatrix}$. Find $3C$.

2. _____

3. Let $D = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$. Find $D - E$.

3. _____

4. Let $F = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 5 & 6 \end{bmatrix}$. Find F^T.

4. _____

5. Let $G = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Express G^T in terms of a, b, c, d.

5. _____

6. Let $H = \begin{bmatrix} 7 & -2 & 3 \end{bmatrix}$. Find H^T

6. _____

7. Given H as in question 6 and F as in question 4, compute FH^T.

7. _____

8. Let $I = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $J = \begin{bmatrix} -1 & 0 \\ 1 & 3 \end{bmatrix}$. Find IJ.

8. _____

9. Let $K = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix}$ and $L = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$. Find KL^T. How does this calculation relate to what we worked on last week?

9. _____

10. Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $N = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$. Express MN in terms of a, b, c, d, e, f, g, h.

10. _____

11. Let $P = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$. Circle the matrix multiplications which are well-defined:

$$PQ \quad QP \quad PQ^T \quad P^TQ \quad Q^TP$$

If any of these matrix multiplications are well-defined, compute them.

2   Matrices and Dynamics of Vectors

12. Draw a matrix diagram for the following matrix model:

$$L = \begin{bmatrix} 0 & 2.4 & 5 \\ 0.5 & 0 & 0 \\ 0 & 0.8 & 0.7 \end{bmatrix}$$

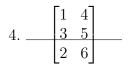# 1   Matrix Algebra

1. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$. Find $A + B$.
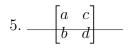
1. $\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}$

2. Let $C = \begin{bmatrix} 3 & -5 & 2 \\ 1 & 0 & 4 \end{bmatrix}$. Find $3C$.

2. $\begin{bmatrix} 9 & -15 & 6 \\ 3 & 0 & 12 \end{bmatrix}$

3. Let $D = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$. Find $D - E$.
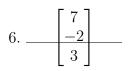
3. $\begin{bmatrix} -1 & 1 \\ 3 & -5 \end{bmatrix}$

4. Let $F = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 5 & 6 \end{bmatrix}$. Find F^T.
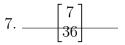
4. $\begin{bmatrix} 1 & 4 \\ 3 & 5 \\ 2 & 6 \end{bmatrix}$

5. Let $G = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Express G^T in terms of a, b, c, d.

5. $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

6. Let $H = \begin{bmatrix} 7 & -2 & 3 \end{bmatrix}$. Find H^T

6. $\begin{bmatrix} 7 \\ -2 \\ 3 \end{bmatrix}$

7. Given H as in question 6 and F as in question 4, compute FH^T.

7. $\begin{bmatrix} 7 \\ 36 \end{bmatrix}$

8. Let $I = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $J = \begin{bmatrix} -1 & 0 \\ 1 & 3 \end{bmatrix}$. Find IJ.

8. $\dfrac{\begin{bmatrix} 1 & 1 \\ 1 & 12 \end{bmatrix}}{}$

9. Let $K = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix}$ and $L = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$. Find KL^T. How does this calculation relate to what we worked on last week?

9. **It's the dot product:** 5

10. Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $N = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$. Express MN in terms of a, b, c, d, e, f, g, h.

10. $\begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$

11. Let $P = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$. Circle the matrix multiplications which are well-defined:

$$PQ \quad QP \quad PQ^T \quad P^TQ \quad Q^TP$$

If any of these matrix multiplications are well-defined, compute them.

Solution:

$$PQ = \begin{bmatrix} 5 & 6 \\ 8 & 7 \end{bmatrix}$$

$$QP = \begin{bmatrix} 4 & 5 & 5 \\ 5 & 4 & 7 \\ 3 & 3 & 4 \end{bmatrix}$$

The products PQ^T, P^TQ, and Q^TP are not well-defined.

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2   Matrices and Dynamics of Vectors

12. Draw a matrix diagram for the following matrix model:

$$L = \begin{bmatrix} 0 & 2.4 & 5 \\ 0.5 & 0 & 0 \\ 0 & 0.8 & 0.7 \end{bmatrix}$$

Solution: