1 Matrices and Dynamics of Vectors

Difference equations

1. Life cycle stages
A freshwater lake is inhabited by a type of amphibian. The amphibians in the lake have three distinct life stages: tadpoles, juveniles, and adults. Each year:

   - 50% of tadpoles mature and become juveniles.
   - 40% of juveniles mature and become adults.
   - Adults lay eggs, and from these eggs, 100 new tadpoles are hatched per adult.

   a. Obtain a system of difference equations to model the change in population of each life stage of the amphibian over time.

   b. Represent the system of difference equations above as a matrix model.

   c. Draw a matrix diagram representing the transition between life cycle stages.

   d. Find the matrix model corresponding to the population change over the course of two years.

2. Disease stages
A clinic observes patients with a certain disease that has three stages: mild, moderate, and severe. Every month:

   - 40% of patients with mild symptoms progress to the moderate stage.
   - 20% of patients with moderate symptoms regress to the mild stage, while another 30% progress to the severe stage.
   - 10% of patients with severe symptoms regress to the moderate stage.

a. Obtain a system of difference equations to model the change in population of each life stage of the amphibian over time.

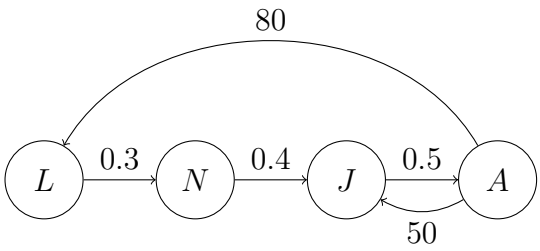
b. Represent the system of difference equations above as a matrix model.

c. Draw a matrix diagram representing the transition between life cycle stages.

3. Draw a matrix diagram for the following matrix model:

$$L = \begin{bmatrix} 0 & 2.4 & 5 \\ 0.5 & 0 & 0 \\ 0 & 0.8 & 0.7 \end{bmatrix}$$

4. Write the matrix model corresponding to the following matrix diagram.



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   - 50% of tadpoles mature and become juveniles.
   - 40% of juveniles mature and become adults.
   - Adults lay eggs, and from these eggs, 100 new tadpoles are hatched per adult.

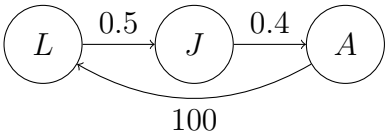
   a. Obtain a system of difference equations to model the change in population of each life stage of the amphibian over time.

$$T_{t+1} = 100A_t$$
$$J_{t+1} = 0.5T_t$$
$$A_{t+1} = 0.4J_t$$

   b. Represent the system of difference equations above as a matrix model.

$$\begin{bmatrix} 0 & 0 & 100 \\ 0.5 & 0 & 0 \\ 0 & 0.4 & 0 \end{bmatrix}$$

   c. Draw a matrix diagram representing the transition between life cycle stages.



   d. Find the matrix model corresponding to the population change over the course of two years.

   It is the square of the matrix from b.

$$\begin{bmatrix} 0 & 40 & 0 \\ 0 & 0 & 50 \\ 0.2 & 0 & 0 \end{bmatrix}$$

2. Disease stages
A clinic observes patients with a certain disease that has three stages: mild, moderate, and severe. Every month:

   - 40% of patients with mild symptoms progress to the moderate stage.

- 20% of patients with moderate symptoms regress to the mild stage, while another 30% progress to the severe stage.
- 10% of patients with severe symptoms regress to the moderate stage.

a. Obtain a system of difference equations to model the change in population of each life stage of the amphibian over time.

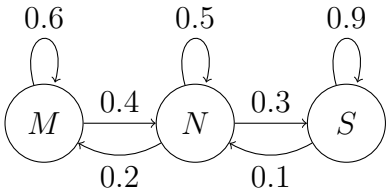
Let M stand for mild, N for moderate, and S for severe.

$$\begin{aligned} M_{t+1} &= 0.6M_t + 0.2N_t \\ N_{t+1} &= 0.4M_t + 0.5N_t + 0.1N_t \\ S_{t+1} &= 0.3N_t + 0.9S_t \end{aligned}$$

b. Represent the system of difference equations above as a matrix model.

$$\begin{bmatrix} 0.6 & 0.2 & 0 \\ 0.4 & 0.5 & 0.1 \\ 0 & 0.3 & 0.9 \end{bmatrix}$$

c. Draw a matrix diagram representing the transition between life cycle stages.



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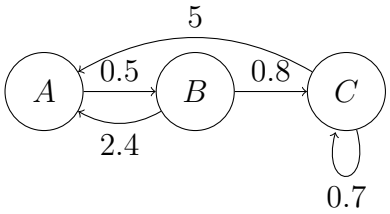
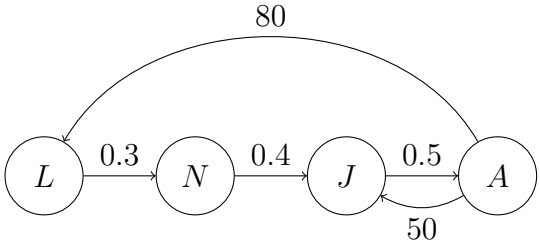


4. Write the matrix model corresponding to the following matrix diagram.



$$\begin{bmatrix} 0 & 0 & 0 & 80 \\ 0.3 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 50 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}$$