

1. Solve for x .

(a) $\log_2(x) = 5$

(b) $e^{x-1} = 3$

(c) $\log_2(x^2 - 1) = 3$

(d) $2^{2^x} = 256$

2. Let $f(x) = 6 \log_2(x - 1) - 3$.

(a) Find the domain and the range of $f(x)$.

(b) Find all zeros of $f(x) = 0$.

(c) What is the inverse function of $f(x)$?

3. Write the first five terms of the sequence defined by $a_n = n^2 + 1$.

4. Consider the recursive sequence $a_1 = 1$ and $a_{n+1} = 2a_n + 1$. Write down the first five terms of this sequence. Can you guess the general formula for a_n ?

1. Solve for x .

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(a) $x = 2^5 = 32$

(b) $x - 1 = \ln 3, x = \ln 3 + 1$

(c) $x^2 - 1 = 2^3 = 8, x = \pm 3$

(d) $2^x = \log_2 256 = 8, x = \log_2 8 = 3$

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(a) Find the domain and the range of $f(x)$.

(b) Find all zeros of $f(x) = 0$.

(c) What is the inverse function of $f(x)$?

(a) Domain: $x > 1$, Range: \mathbb{R}

(b) $6 \log_2(x - 1) - 3 = 0 \Leftrightarrow \log_2(x - 1) = \frac{1}{2} \Leftrightarrow x - 1 = 2^{1/2} = \sqrt{2} \Leftrightarrow x = 1 + \sqrt{2}$

(c) $x = 6 \log_2(y - 1) - 3 \Leftrightarrow \log_2(y - 1) = \frac{x+3}{6} \Leftrightarrow y - 1 = 2^{(x+3)/6} \Leftrightarrow y = 1 + 2^{(x+3)/6} = f^{-1}(x)$

3. Write the first five terms of the sequence defined by $a_n = n^2 + 1$.

2, 5, 10, 17, 26

4. Consider the recursive sequence $a_1 = 1$ and $a_{n+1} = 2a_n + 1$. Write down the first five terms of this sequence. Can you guess the general formula for a_n ?

The first terms are 1, 3, 7, 15, 31. You *may* guess that (you can compute more terms if you want) $a_n = 2^n - 1$. Try to prove this yourself, by defining a new sequence $b_n = a_n + 1$ (what is the recursion formula that b_n satisfies?).