1. Compute the derivatives the following functions.
   (a) $f(x) = 20231010$
   (b) $f(x) = x^2 + 2x + 3$
   (c) $f(x) = \sqrt{x+4} - \sqrt{x+1}$
   (d) $f(x) = x/(x^2 + 4)$
   (e) $f(x) = 1/(1 + e^{-x})$
   (f) $f(x) = e^x \cos(x)$

2. By using the definition of the derivative, compute the derivative of $f(x) = \sqrt{x}$.

3. Let $f(x) = xe^x$. Compute $f'(x)$, $f''(x)$, and $f'''(x)$. Can you guess what $f^{(n)}(x)$ is in general?

4. What is $\frac{d^{2023}}{dx^{2023}}(e^{2x})$?

5. Find the equation of the tangent line of $y = \ln(x)\ln(x+1)$ at $x = 2$.

6. Show that $y = 6x^3 + 5x - 3$ has no tangent line of slope 4. What is the minimum possible slope of tangent lines?

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   (f) $f(x) = e^x \cos(x)$

   (a) 0
   (b) $2x + 2$
   (c) $\frac{1}{2\sqrt{x+4}} - \frac{1}{2\sqrt{x+1}}$
   (d) $\frac{4-x^2}{(x^2+4)^2}$
   (e) $\frac{e^{-x}}{(1+e^{-x})^2}$
   (f) $e^x(\cos(x) - \sin(x))$

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2. By using the definition of the derivative, compute the derivative of $f(x) = \sqrt{x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$
$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

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3. Let $f(x) = xe^x$. Compute $f'(x)$, $f''(x)$, and $f'''(x)$. Can you guess what $f^{(n)}(x)$ is in general?

Use product rule. $f'(x) = (x+1)e^x$, $f''(x) = (x+2)e^x$, $f'''(x) = (x+3)e^x$. In general, we can guess that $f^{(n)}(x) = (x+n)e^x$. This can be proven by mathematical induction. (It holds for $n = 1$. Assume that $f^{(n)}(x) = (x+n)e^x$ for some n. Then $f^{(n+1)}(x) = \frac{d}{dx} f^{(n)}(x) = (x+n+1)e^x$, so it is also true for $n+1$ and we are done.)

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4. What is $\frac{d^{2023}}{dx^{2023}}(e^{2x})$?

First, by applying product rule to $e^{2x} = e^x \cdot e^x$, we can check that $\frac{d}{dx}(e^{2x}) = 2e^{2x}$. By continuting differentiation, we get $\frac{d^2}{dx^2}(e^{2x}) = 2^2 e^{2x}$, and in general we have $\frac{d^n}{dx^n}(e^{2x}) = 2^n e^{2x}$. Hence $\frac{d^{2023}}{dx^{2023}}(e^{2x}) = 2^{2023} e^{2x}$.

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5. Find the equation of the tangent line of $y = \ln(x)\ln(x+1)$ at $x = 2$.

By product rule, we have

$$\frac{dy}{dx} = \frac{1}{x}\ln(x+1) + \ln(x)\frac{1}{x+1}$$

so the slope of the tangent line at $x = 2$ is $\frac{\ln(3)}{2} + \frac{\ln(2)}{3}$. Since it passes $(2, \ln(2)\ln(3))$, the equation of the line is

$$y = \left(\frac{\ln(3)}{2} + \frac{\ln(2)}{3}\right)(x-2) + \ln(2)\ln(3).$$

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6. Show that $y = 6x^3 + 5x - 3$ has no tangent line of slope 4. What is the minimum possible slope of tangent lines?

The derivative of the function is

$$\frac{dy}{dx} = 18x^2 + 5$$

and since $18x^2 \geq 0$ for any x, we have $\frac{dy}{dx} \geq 5$. Hence the minimum possible slope of the tangent line is 5 (at $x = 0$), and there's no tangent line of slope 4.
