Derivatives of Logarithmic and Inverse Tangent Functions

1. Compute the derivative of $y = \ln(x^2 + 1)$.

2. Differentiate the function $f(x) = x\ln(x) - x$.

3. Find the derivative of $g(x) = \arctan(3x^2)$.

4. Given $h(x) = \ln(x) + \arctan(x)$, find $h'(x)$.

Linear Approximations, Newton's Method, and Taylor Polynomials

1. Find the linear approximation of the function $f(x) = \sqrt{x}$ at $x = 4$.

2. Using Newton's method, estimate the root of the equation $x^3 - x - 2 = 0$ given an initial approximation of $x_0 = 2$.

3. Write down the second-degree Taylor polynomial for $f(x) = \ln(x)$ centered at $x = 1$.

Maximum and Minimum Values

1. Use the Extreme Value Theorem to determine if the function $f(x) = x^4 - 4x^3$ has an absolute maximum and minimum on the interval $[0, 4]$.

2. Using Fermat's Theorem, find the local maximum and minimum values of the function $g(x) = x^3 - 3x^2 + 2$.

3. Apply the Closed Interval Method to find the absolute maximum and minimum values of the function $h(x) = x^2 - 4x + 5$ on the interval $[-2, 5]$.

Derivatives of Logarithmic and Inverse Tangent Functions

1. Compute the derivative of $y = \ln(x^2 + 1)$.

   Solution: Using the chain rule, $y' = \frac{2x}{x^2+1}$

2. Differentiate the function $f(x) = x \ln(x) - x$.

   Solution: Using the product rule, $f'(x) = \ln(x) + 1 - 1 = \ln(x)$

3. Find the derivative of $g(x) = \arctan(3x^2)$.

   Solution: Using the chain rule, $g'(x) = \frac{6x}{1+(3x^2)^2}$

4. Given $h(x) = \ln(x) + \arctan(x)$, find $h'(x)$. **Solution:** $h'(x) = \frac{1}{x} + \frac{1}{1+x^2}$

Linear Approximations, Newton's Method, and Taylor Polynomials

1. Find the linear approximation of the function $f(x) = \sqrt{x}$ at $x = 4$. **Solution:** $L(x) = f(4) + f'(4)(x - 4) = 2 + \frac{1}{4}(x - 4)$

2. Using Newton's method, estimate the root of the equation $x^3 - x - 2 = 0$ given an initial approximation of $x_0 = 2$. **Solution:** Using the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ and the given function, the next approximation x_1 would be $x_1 \approx 1.3$ (Rounded off after one iteration for simplicity.)

3. Write down the second-degree Taylor polynomial for $f(x) = \ln(x)$ centered at $x = 1$. **Solution:** $P_2(x) = (x - 1) - \frac{(x-1)^2}{2}$

Maximum and Minimum Values

1. Use the Extreme Value Theorem to determine if the function $f(x) = x^4 - 4x^3$ has an absolute maximum and minimum on the interval $[0, 4]$. **Solution:** By differentiating and

   setting the derivative to zero, we find $x = 0$ and $x = 3$ as critical points. By evaluating $f(x)$ at these points and the endpoints of the interval, we see that the function attains its absolute maximum of 16 at $x = 4$ and its absolute minimum of -27 at $x = 3$.

2. Find the local maximum and minimum values of the function $g(x) = x^3 - 3x^2 + 2$. **Solution:** The derivative is $g'(x) = 3x^2 - 6x$. Setting this equal to zero gives $x = 0$

   and $x = 2$ as critical points. The second derivative is $g''(x) = 6x - 6$, which is positive at $x = 0$ and negative at $x = 2$. Hence, $x = 0$ is a local minimum and $x = 2$ is a local maximum.

3. Find the absolute maximum and minimum values of the function $h(x) = x^2 - 4x + 5$ on the interval $[-2, 5]$. **Solution:** The derivative is $h'(x) = 2x - 4$. Setting this equal

   to zero gives $x = 2$ as the only critical point. Evaluating $h(x)$ at this point and the endpoints of the interval, the function attains its absolute maximum of 9 at $x = -2$ and its absolute minimum of 1 at $x = 2$.