1. Evaluate following indefinite/definite integrals.
   (a) $\int \frac{2x}{\sqrt{1-x^2}}\,dx$
   (b) $\int x^2 e^{x^3}\,dx$
   (c) $\int x^2 e^x\,dx$
   (d) $\int \frac{1}{x(2x-1)}\,dx$
   (e) $\int_0^1 \frac{x^2}{x+1}\,dx$
   (f) $\int_0^1 \frac{2x}{1+x^2}\,dx$
   (g) $\int_{-1/2}^{1/2} \frac{\sin(\pi x)}{\sqrt{1-4x^2}}\,dx$
   (h) $\int_0^1 \frac{x^2-x}{(x+1)(x^2+1)}\,dx$
   (i) $\int_1^{\sqrt{3}} \arctan(1/x)\,dx$

2. Assume that a twice differentiable function $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(1) = 2, f(2) = 0, f'(1) = 4, f'(2) = -1$. Evaluate the following integrals:
   (a)
   $$\int_1^2 f''(x)dx.$$
   (b)
   $$\int_1^2 xf''(x)dx.$$
   (c)
   $$\int_1^2 f'(x)f''(x)dx.$$

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(a) Using the ubstitution $u = 1 - x^2$ gives $du = -2xdx$ and

$$\int \frac{2x}{\sqrt{1-x^2}}dx = \int \frac{-du}{\sqrt{u}} = -2\sqrt{u} + C = -2\sqrt{1-x^2} + C.$$

(b) Using the substitution $u = x^3$, $du = 3x^2dx$. We get $\int \frac{1}{3}e^u du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$.

(c) Use integration by parts twice.

$$\int x^2e^xdx = x^2e^x - \int 2xe^xdx = x^2e^x - \left(2xe^x - \int 2e^xdx\right) = x^2e^x - 2xe^x - 2e^x + C.$$

(d) Use partial fraction.

$$\int \frac{1}{x(2x-1)}dx = \int \left(\frac{2}{2x-1} - \frac{1}{x}\right)dx = \ln|2x-1| - \ln|x| + C.$$

(e) Using the substitution $u = x+1$ gives

$$\int_1^2 \frac{(u-1)^2}{u}du = \int_1^2 u - 2 + \frac{1}{u}du = \left[\frac{1}{2}u^2 - 2u + \ln|u|\right]_1^2 = -\frac{1}{2} + \ln 2.$$

(f) $[\ln(1+x^2)]_0^1 = \ln 2$.

(g) The function is an odd function, so the integral over the symmetric domain is 0.

(h) Use partial fraction.

$$\int_0^1 \frac{x^2-x}{(x+1)(x^2+1)}dx = \int_0^1 \frac{1}{x+1} - \frac{1}{x^2+1}dx = [\ln|x+1| - \arctan(x)]_0^1 = \ln 2 - \frac{\pi}{4}.$$

(i) Use integration by parts.

$$\begin{aligned}\int_1^{\sqrt{3}} \arctan(1/x)dx &= [x\arctan(1/x)]_1^{\sqrt{3}} - \int_1^{\sqrt{3}} x\frac{-1/x^2}{1+1/x^2}dx \\ &= \frac{\sqrt{3}\pi}{6} - \frac{\pi}{4} + \int_1^{\sqrt{3}} \frac{xdx}{1+x^2} \\ &= \frac{\sqrt{3}\pi}{6} - \frac{\pi}{4} + \left[\frac{1}{2}\ln(1+x^2)\right]_1^{\sqrt{3}} = \frac{\sqrt{3}\pi}{6} - \frac{\pi}{4} + \frac{\ln 2}{2}.\end{aligned}$$

2. Assume that a twice differentiable function $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(1) = 2, f(2) = 0$, $f'(1) = 4, f'(2) = -1$. Evaluate the following integrals:

(a)
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(b)
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(c)
$$\int_1^2 f'(x)f''(x)dx.$$

(a) By the fundamental theorem of calculus, it is $f'(2) - f'(1) = -5$.

(b) Using integration by parts, we get

$$\int_1^2 xf''(x)dx = [xf'(x)]_1^2 - \int_1^2 f'(x)dx = 2f'(2) - f'(1) - (f(2) - f(1)) = -8.$$

(c) Using integration by parts, we get

$$\int_1^2 f'(x)f''(x)dx = [f'(x)f'(x)]_1^2 - \int_1^2 f''(x)f'(x)dx = -3 - \int_1^2 f'(x)f''(x)dx$$

which implies $2\int_1^2 f'(x)f''(x)dx = 3$, so $\int_1^2 f'(x)f''(x)dx = \frac{3}{2}$. You can also use $(\frac{1}{2}f'(x)^2)' = f'(x)f''(x)$ to integrate it directly.