1. Compute the integrals.
   (a) $\int_0^\infty \frac{e^x}{e^{2x}+1} dx$
   (b) $\int_0^\infty \frac{x}{(1+x^2)^2} dx$

2. Find the area of the region bounded by the curves $y = x^2$ and $y = 4x - x^2$.

3. Let $f(x) = e^x$ and $g(x) = x^2 e^x$.

   (a) Find intersection points of the graphs $y = f(x)$ and $y = g(x)$.

   (b) Find the area of the region enclosed by the curves.

4. Let a be the number such that the line $x = a$ bisects the area under the curve $y = \ln x$, $1 \leq x \leq e$. Find the x-intercept of the tangent line of $y = \ln x$ at $x = a$.

1. Compute the integrals.
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(a) Using the substitution $u = e^x$, $du = e^x dx$ gives

$$\int_0^\infty \frac{e^x}{e^{2x}+1}dx = \int_1^\infty \frac{1}{u^2+1}du = [\arctan(u)]_1^\infty = \frac{\pi}{4}.$$

(b) Using the substitution $u = x^2$, $du = 2xdx$ gives

$$\int_0^\infty \frac{x}{(1+x^2)^2}dx = \int_0^\infty \frac{1}{2}\frac{1}{(1+u)^2}du = \left[-\frac{1}{2}\frac{1}{1+u}\right]_0^\infty = \frac{1}{2}.$$

2. Find the area of the region bounded by the curves $y = x^2$ and $y = 4x - x^2$.

Let's find the intersection points first. By solving $x^2 = 4x - x^2$, we get $x = 0$ or $x = 2$, and the intersection points are $(0,0)$ and $(2,4)$. For $0 \leq x \leq 2$, we have $4x - x^2 \geq x^2$, so the area of the region becomes

$$\int_0^2 (4x - x^2) - x^2 dx = \int_0^2 4x - 2x^2 dx = \left[2x^2 - \frac{1}{3}x^3\right]_0^2 = \frac{8}{3}.$$

3. Let $f(x) = e^x$ and $g(x) = x^2e^x$.
   (a) Find intersection points of the graphs $y = f(x)$ and $y = g(x)$.
   (b) Find the area of the region enclosed by the curves.

(a) By solving $e^x = x^2e^x$, we get $x = -1, 1$.

(b) For $-1 \leq x \leq 1$, $f(x) \geq g(x)$ and the area becomes

$$\begin{aligned}
\int_{-1}^{1} e^x - x^2e^x dx &= \int_{-1}^{1} (1-x^2)e^x dx \\
&= \left[(1-x^2)e^x\right]_{-1}^{1} - \int_{-1}^{1} (-2x)e^x dx \\
&= \int_{-1}^{1} 2xe^x dx \\
&= [2xe^x]_{-1}^{1} - \int_{-1}^{1} 2e^x dx \\
&= (2e + 2e^{-1}) - [2e^x]_{-1}^{1} \\
&= 2e + 2e^{-1} - (2e - 2e^{-1}) = 4e^{-1}.
\end{aligned}$$

Here we used integration by parts twice.

4. Let a be the number such that the line $x = a$ bisects the area under the curve $y = \ln x$, $1 \leq x \leq e$. Find the x-intercept of the tangent line of $y = \ln x$ at $x = a$.

The number a satisfies

$$\int_1^a \ln x dx = \int_a^e \ln x dx.$$

By applying integration by parts, $x \ln x - x$ is an antiderivative of $\ln x$ and we have

$$a \ln a - a - (-1) = 0 - (a \ln a - a) \Leftrightarrow a \ln a - a = -\frac{1}{2}.$$

(You don't need to solve the equation - actually you can't.) How the equation of the tangent line at $x = a$ is

$$y - \ln a = \frac{1}{a}(x - a),$$

so the x-intercept of the line is the zero of

$$0 - \ln a = \frac{1}{a}(x - a) \Leftrightarrow x = -(a \ln a - a),$$

which equals to $\frac{1}{2}$ by the previous computation.