More Jordan forms

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1. Let $T \in \mathcal{L}(\mathbb{C}^3)$ be a matrix given by

$$\begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

where a is 0 or 1. Fill the blanks in the table below.

a	min. poly.	char. poly.	$\dim E(1,T)$	$\dim G(1,T)$	$\dim E(2,T)$	$\dim G(2,T)$
0						
1						

2. Here's a Jordan form of some $T \in \mathcal{L}(\mathbb{C}^{10})$:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   (a) What are the eigenvalues?

   (b) What is the characteristic polynomial?

   (c) What is the minimal polynomial?

   (d) Find dimension of $E(\lambda, T)$ for each eigenvalue λ.

   (e) Find dimension of $G(\lambda, T)$ for each eigenvalue λ.

3. Let $T \in \mathcal{L}(\mathbb{C}^4)$. Assume that T has two eigenvalues $-2, 1$.
   (a) Find all the possible pairs of (minimal polynomial, characteristic polynomial). How many are there?
   (b) Which pair gives diagonalizable T?
   (c) Find Jordan form for each pairs. Check that there's only one possible Jordan forms (up to permuting Jordan blocks) for each pair.
4. Find two different 5 by 5 Jordan forms with same characteristic polynomial and minimal polynomial. Here we say that two Jordan forms are indifferent if they are the same up to permutation of Jordan blocks.
5. Find a 6 by 6 Jordan form T where
   - it has two distinct eigenvalues $2, 3$,
   - its minimal polynomial has degree 4,
   - its characteristic polynomial is $(z-2)^3(z-3)^3$,
   - two eigenspaces $E(2, T)$ and $E(3, T)$ have different dimensions,
   - $(T - 2I)^k$ becomes a (genuine) diagonal matrix for some $k > 0$.