Week 1, August 30

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1. (a) Read and digest the proof of the Theorem 1.27 in the book again. Which axioms are used? Which are *not* used?

   (b) Now, do the exercise 1A 5 and 6, if you haven't done yet. Then read the proof of 1.27 again. Did you find something interesting/satisfying?

   (c) (*) You may learn several "algebraic structures" in a future (at least if you take Math 113). Here is the first concept you will learn:

   A set G with a binary operation $* : G \times G \to G$ is called a *group* if it satisfies three axioms:

   i. (associativity) $(a * b) * c = a * (b * c)$ for any $a, b, c \in G$.

   ii. (identity) There is $e \in G$ such that $a * e = e * a = a$ for any $a \in G$.

   iii. (inverse) For any $a \in G$, there exists $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$.

   Now, prove the following: for any $a \in G$, there exists a *unique* inverse of a. Even if you see the definition of a group for the first time in your lift, I believe you already know how to prove.

2. Show that the set of 2 by 2 matrices over real numbers form a vector space over $\mathbb{R}$ with the "natural" choice of $+$ and $\cdot$. You may need to check all the axioms of vector spaces. (Maybe boring, but worths to do.)

3. (a) Let $\mathbb{R}_+$ be a set of positive real numbers with usual additions and $\mathbb{R}$-multiplications. Is it a vector space? Why?

   (b) Now, consider the same set again, but with weird additions and multiplications: for $x, y \in \mathbb{R}_+$ and $a \in \mathbb{R}$,

   - (addition) $x \oplus y = xy$
   - (multiplication) $a \odot x = x^a$

   Prove that $(\mathbb{R}_+, \oplus, \odot)$ is a vector space over $\mathbb{R}$. What are the additive and multiplicative identities?