1. (Sets) Let $A = \{a,b,c,d,e,f\}$ and $B = \{b,e,f,h,j\}$. Describe the following sets.
   (a) $A \cap B$
   (b) $A \cup B$
   (c) $A \backslash B$
   (d) $B \backslash A$
   (e) $(A \backslash B) \cap (B \backslash A)$
   (f) $(A \backslash B) \cup (B \backslash A)$

2. (Countings) Count the followings.
   (a) How many functions from $\{1,2,3\}$ to $\{2,4,6\}$?
   (b) There are 40 and 50 graduate students in 9th and 10th floor Evans hall, respectively. In how many ways can one representative be picked who is either on the 9th floor or 10th floor?
   (c) Same situation as (b), but what if we pick one representative on each floor?
   (d) How many bit strings are there of length six?
   (e) How many 5-element DNA sequences that start with T?

3. (More countings) Count the followings.
   (a) How many different ways to color four tires on a car with four different colors?
   (b) How many four-letter strings that can be made with lowercase letters?
   (c) Same as (b), but what if all the letters are required to be different?
   (d) Same as (b), but what if all the letters are required to be the same?
   (e) Suppose you toss six coins. How many ways are there to obtain exactly three heads?
   (f) Same as (e), but what if there are at least three heads?
   (g) Estimate the product of number of hairs of each person in UC Berkeley.

4. [1] Define a sequence of sets A_n, $n \geq 0$ as

$$A_0 = \varnothing, \quad A_{n+1} = \{A_n\} \cup A_n.$$

   (a) Find A_1, A_2, and A_3. What are their sizes?
   (b) What is $|A_{2024}|$?

[1] This is how mathematicians define natural numbers.

1. (Sets) Let $A = \{a,b,c,d,e,f\}$ and $B = \{b,e,f,h,j\}$. Describe the following sets.
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   (a) $\{b,e,f\}$
   (b) $\{a,b,c,d,e,f,h,j\}$
   (c) $\{a,c,d\}$
   (d) $\{h,j\}$
   (e) $\varnothing$ (This is always empty set, no matter what A and B are. Can you explain why?)
   (f) $\{a,c,d,h,j\}$

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   (a) $3^3 = 27$
   (b) $40 + 50 = 90$
   (c) $40 \times 50 = 2000$
   (d) $2^6 = 64$
   (e) $4^4 = 256$

3. (More countings) Count the followings.
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   (g) Estimate the product of number of hairs of each person in UC Berkeley.

   (a) $4! = 24$
   (b) 26^4
   (c) $26 \cdot 25 \cdot 24 \cdot 23$
   (d) 26 (Choose the letter)
   (e) ${}_6C_3 = 20$
   (f) ${}_6C_3 + {}_6C_4 + {}_6C_5 + {}_6C_6 = 20 + 15 + 6 + 1 = 42$
   (g) There is a bald person with 0 hairs, so it is 0.

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   (a) Find A_1, A_2, and A_3. What are their sizes?
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   (a) $A_1 = \{\varnothing\} \cup \varnothing = \{\varnothing\}$, $A_2 = \{\{\varnothing\}\} \cup \{\varnothing\} = \{\{\varnothing\}, \varnothing\}$, $A_3 = \{\{\{\varnothing\}, \varnothing\}\} \cup \{\{\varnothing\}, \varnothing\} = \{\{\{\varnothing\}, \varnothing\}, \{\varnothing\}, \varnothing\}$. The sizes are $1, 2, 3$, respectively.
   (b) The recurrence relation tells you that A_{n+1} is A_n with one more extra *element* A_n. Hence the size increases by one for each step, and we have $|A_{2024}| = 2024$ (and $|A_n| = n$ in general).

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