1. (a) How many ways are there to assign three jobs to five employees if each employee can be given more than one job?
   (b) How many different strings can be formed using all the letters in MISSISSIPPI?
   (c) Same as (b), but only using ten of them?
   (d) How many terms you get when you expand $(x+y+z)^7$?

   (a) 5^3
   (b) $\frac{11!}{4!4!2!}$
   (c) Divide into several cases depending on which character would be missed:
   - Misses one M: $\frac{10!}{4!4!2!}$
   - Misses one I: $\frac{10!}{3!4!2!}$
   - Misses one S: $\frac{10!}{4!3!2!}$
   - Misses one P: $\frac{10!}{4!4!}$

   so the total number of strings is $\frac{10!}{4!4!2!} + \frac{10!}{3!4!2!} + \frac{10!}{4!3!2!} + \frac{10!}{4!4!}$
   (d) ${}_{3+7-1}C_7 = {}_9C_7 = 36$.

2. (a) How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 10$ where x_1, x_2, x_3, x_4 are nonnegative integers?
   (b) Same as (a), but what if we also require $x_1 \geq 3$?
   (c) Same as (a), but what if we also require $x_2 < 5$?
   (d) Satisfying (b) and (c)?
   (e) Satisfying (b) or (c)?

   (a) ${}_{4+10-1}C_{10} = {}_{13}C_{10}$
   (b) Consider a new equation $x_1' + x_2 + x_3 + x_4 = (x_1 - 3) + x_2 + x_3 + x_4 = 7$, where $x_1' = x_1 - 3 \geq 0$. This gives ${}_{4+7-1}C_7 = {}_{10}C_7$.
   (c) Consider the complement. The number of solutions with $x_2 \geq 5$ is ${}_{4+5-1}C_5 = {}_8C_5$, hence the number of solutions with $x_2 < 5$ is ${}_{13}C_{10} - {}_8C_5$.
   (d) Combination of (b) and (c). Count the number of nonnegative integer solutions of the new equation $x_1' + x_2 + x_3 + x_4 = 7$ with $x_2 < 5$, and we can use complement, which gives ${}_{4+7-1}C_7 - {}_{4+2-1}C_2 = {}_{10}C_7 - {}_5C_2$.
   (e) Principle of Inclusion and Exclusion. (b) + (c) - (d).

3. (a) Two dice are rolled. What is the probability of
   i. both numbers are one?
   ii. both numbers are even?
   iii. sum of two numbers are less then six?
   (b) What would be your best strategy if you are gambling with two dice and need to guess the total outcome?

(a) $\frac{1}{36}$

(b) $\frac{9}{36} = \frac{1}{4}$

(c) $\frac{1+2+3+4+5}{36} = \frac{15}{36} = \frac{5}{12}$

The most likely event to happen is when the summation is 7, with the probability $\frac{6}{36} = \frac{1}{6}$.

4. A fair coin tossed 10 times. Which one is more likely to happen:
   (a) 5 heads and 5 tails,
   (b) something else.

   What if you toss a coin 100 times and ask the same question with 50/50?

There are ${}_{10}C_5 = 252$ possible cases for (a) among total 2^{10} possiblilities, and the probability is $\frac{{}_{10}C_5}{2^{10}} = 0.246....$ Hence (b) is more likely to happen. Similarly, for 100 coin tosses, the probability for having exactly half heads and tails is

$$\frac{{}_{100}C_{50}}{2^{100}} = 0.00795...$$

so it is much more unlikely to happen.