# Conditional Probability

**1.**

Twenty-five students go to a waterpark on an end-of-year fieldtrip. 10 wear a red swimsuit (7 of whom end up with a sunburn), 4 wear a green swimsuit (1 of whom ends up with a sunburn), 7 wear a blue swimsuit (3 of whom end up with a sunburn) and 4 wear a multicolor swimsuit (2 of whom end up with a sunburn). We choose a student at random.

(a) What is the probability the student has a sunburn given then have a blue swimsuit?

(b) What is the probability the student is wearing a green swimsuit given they have a sunburn?

(c) What is the probability the student is not wearing a mutlicolor swimsuit given they do not have a sunburn?

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| Swimsuits and Sunburns | | |
|---|---|---|
| | Sunburn | No Sunburn |
| Red | 7 | 3 |
| Blue | 3 | 4 |
| Green | 1 | 3 |
| Multi | 2 | 2 |

1. $$\frac{P(\text{sun} \cap \text{blue})}{P(\text{blue})} = \frac{3/25}{7/25}$$

2. $$\frac{P(\text{green} \cap \text{sun})}{P(\text{sun})} = \frac{1/25}{13/25}$$

3. $$\frac{P(\text{no mutli} \cap \text{no sun})}{P(\text{no sun})} = \frac{10/25}{12/25}$$

**2**

My cat is knocking cups off the counter. There are two blue mugs, two clear mugs, 1 white mug, 4 clear wine glasses, 2 blue wine glasses, and 1 clear priceless heirloom. She knocks two down at random before I am able to stop her.

(a) What is the probability she knocked down at least one blue cup given she knocked down two mugs?

(b) What is the probability she knocked down two mugs given she knocked down at least one blue cup?

(c) What is the probability she knocked down the priceless heirloom given she knocked down exactly one clear cup?

(d) What is the probability she knocked down the priceless heirloom given she knocked down at least one clear cup?

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| Cups | | | |
|---|---|---|---|
| | Blue | Clear | White |
| Mug | 2 | 2 | 1 |
| WG | 2 | 4 | 0 |
| PH | 0 | 1 | 0 |

There are 12 total cups.

1. $$\frac{P(\geq 1 \text{ blue} \cap 2 \text{ mugs})}{P(2\text{mugs})} = \frac{\frac{2}{12}\frac{1}{11} + 2\left(\frac{2}{12}\frac{3}{11}\right)}{\frac{5}{12}\frac{4}{11}}$$

2. $$\frac{P(\geq 1 \text{ blue} \cap 2 \text{ mugs})}{P(\geq 1 \text{ blue})} = \frac{\frac{2}{12}\frac{1}{11} + 2\left(\frac{2}{12}\frac{3}{11}\right)}{\frac{4}{12}\frac{3}{11} + 2\left(\frac{4}{12}\frac{8}{11}\right)}$$

3. $$\frac{P(\text{ PH} \cap 1 \text{ clear})}{P(1 \text{ clear})} = \frac{2\left(\frac{1}{12}\frac{5}{11}\right)}{2\left(\frac{7}{12}\frac{5}{11}\right)}$$

4.

$$\frac{P(\text{ PH}\cap \geq 1 \text{ clear})}{P(\geq 1 \text{ clear})} = \frac{2\left(\frac{1}{12}\frac{6}{11}\right) + 2\left(\frac{1}{12}\frac{5}{11}\right)}{\frac{7}{12}\frac{6}{11} + 2\left(\frac{7}{12}\frac{5}{11}\right)}$$

**3.**

Suppose Buzzfeed creates an "Is Your Boss a Michael Scott?" quiz which has a 28% false positive rate and a 1% false negative rate. Suppose that 3% of bosses are actually Michael Scotts.

(a) What is the probability that a boss labeled "Not a Michael Scott" is indeed Not a Michael Scott?

(b) What is the probability that a boss labeled "Michael Scott" is actually a Michael Scott?

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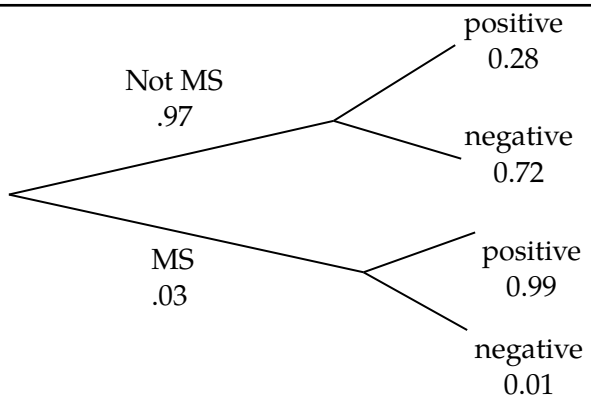


1.

$$P(\text{Not MS}|\text{negative}) = \frac{P(\text{negative}|\text{Not MS})P(\text{Not MS})}{P(\text{ negative})} = \frac{(0.72)(0.97)}{(0.72)(0.97) + (0.01)(0.03)}$$

2.

$$P(\text{MS}|\text{positive}) = \frac{P(\text{positive}|\text{MS})P(\text{MS})}{P(\text{ positive})} = \frac{(0.99)(0.03)}{(0.28)(0.97) + (0.99)(0.03)}$$