1. Boys of a certain age are known to have a mean weight of $\mu = 85$ pounds. A complaint is made that the boys living in a municipal children's home are underfed. As one bit of evidence, $n = 100$ boys (of the same age) are weighed and found to have a mean weight of $x = 80$ pounds. It is known that the population standard deviation σ is 20 pounds.

   (a) Assume that we want to determine whether the average is 85 or smaller. Set the appropriate null hypothesis H_0 and alternative hypothesis H_1. Is this one-sided or two-sided?

   (b) By using Z-statistic, draw a conclusion (reject H_0 or not?). Use the significance level (threshold) as $\alpha = 0.02$. You may need to look up a Z-score table.

   (c) What can we do if the population standard deviation is unknown and the sample size is small?

(a)

$$\begin{cases} H_0 : \mu = 85 \\ H_0 : \mu < 85 \end{cases}$$

This is one-sided.

(b) Assume that the null hypothesis is true. Then the Z-score is

$$Z = \frac{80 - 85}{20/\sqrt{100}} = -2.5.$$

So the p-value is

$$P(Z \leq -2.5) = 1 - z(2.5) = 1 - 0.99379 = 0.00621 < 0.02,$$

and we reject H_0.

(c) We may use T-statistic instead of Z-statistic.

2. Assume that you sampled data $x_1, \ldots, x_n$ from a normal distribution X. Assume that the sample mean of x_i's is 10 If the null and alternative hypothesis are

$$\begin{cases} H_0 : \mu = 10 \\ H_A : \mu \neq 10 \end{cases}$$

What is the p-value? If your threshold was 0.05, would you reject your null hypothesis?

The corresponding Z-statistic is $(10 - 10)/(\sigma/\sqrt{n}) = 0$, regardless of σ and n. Hence the p-value is 1 (largest possible), and we do not reject the null hypothesis.

3. You have a 6-sided die. You suspect that a die is biased toward coming up 1s. Let

$$\begin{cases} H_0 : p = \frac{1}{6} \\ H_A : p > \frac{1}{6} \end{cases}$$

(a) If you get four 1s in five rolls, what can you conclude with significance level $\alpha = 0.05$?

(b) If you get 40 1s in 180 rolls, what can you conclude with the same significance level? You may use Z-statistic.

(a)

$$P(X \geq 4) = \binom{5}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right) + \binom{5}{5}\left(\frac{1}{6}\right)^5 = \frac{26}{6^5} = 0.0033\cdots < 0.05,$$

so we reject H_0.

(b) The Z-statistic is

$$\frac{\frac{40}{180} - \frac{1}{6}}{\frac{1}{\sqrt{180}}\sqrt{\frac{1}{6}\frac{5}{6}}} = 2.$$

Based on the table, the p-value is

$$P(Z \geq 2.0) = 1 - 0.97725 = 0.02275 < 0.05,$$

so we reject H_0.