All the previous worksheets are available in seewoo5.github.io/teaching/2026Spring.
Kewords: Area under a parametric curve, arc length, surface area
Tips: Use Desmos or Wolfram Alpha to visualize parametric curves.

1. Remind what you have learned in single variable calculus (Math 1A/1B).

2. Find an equation of the tangent line to the parametric curve

$$x = 1 + \ln t, \quad y = t^2 + 2$$

   at the point corresponding to $t = 1$. Can you do the same thing in an another way?

3. Find the area enclosed by the x-axis and the curve

$$x = t^3 + 1, \quad y = 2t - t^2$$

4. Find the length of the curve

$$x = e^t \cos t, \quad y = e^t \sin t, \quad 0 \leq t \leq \pi.$$

   What if we replace the interval with $-\infty < t \leq 0$?

5. Enter "Pikachu curve" in Wolfram Alpha.

Math 53 **Worksheet 260123** Spring 2026

1. You know what you have learned!

2. Two methods:

   (1) Differentiate with respect to t and use $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$:

$$dx/dt = 1/t, \quad dy/dt = 2t \implies \left.\frac{dy}{dx}\right|_{t=1} = \frac{2\cdot 1}{1} = 2.$$

   The point is $(x(1), y(1)) = (1, 3)$, so the tangent line is $y - 3 = 2(x-1) \Leftrightarrow y = 2x+1$.

   (2) Solve for t from $x = 1 + \ln t$ to get $t = e^{x-1}$, then $\frac{dy}{dx} = 2e^{2(x-1)}$ and get the same slope at $x = 1$.

3. The curve meets the x-axis when $y = 0$, i.e. $t = 0, 2$. Then the area is

$$A = \int y\,dx = \int_0^2 y(t)x'(t)\,dt = \int_0^2 (2t - t^2)(3t^2)\,dt = \int_0^2 (6t^3 - 3t^4)\,dt = \left[\tfrac{3}{2}t^4 - \tfrac{3}{5}t^5\right]_0^2 = 24 - \tfrac{96}{5} = \tfrac{24}{5}.$$

4. Compute

$$dx/dt = e^t(\cos t - \sin t), \quad dy/dt = e^t(\sin t + \cos t),$$

   so

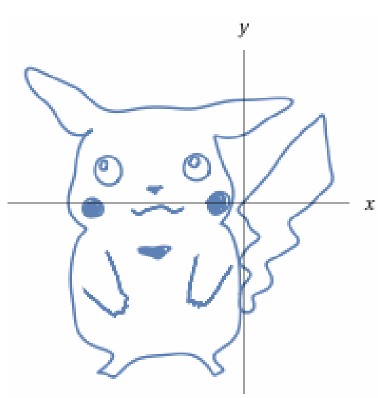
$$(dx/dt)^2 + (dy/dt)^2 = e^{2t}\big((\cos t - \sin t)^2 + (\sin t + \cos t)^2\big) = 2e^{2t}.$$

   Thus

$$L = \int_0^\pi \sqrt{2}e^t\,dt = \sqrt{2}(e^\pi - 1).$$

   For $-\infty < t \le 0$ the same integrand gives (improper integral)

$$L = \int_{-\infty}^0 \sqrt{2}e^t\,dt = \sqrt{2}.$$

5. 

(plotted for t from 0 to 52π)