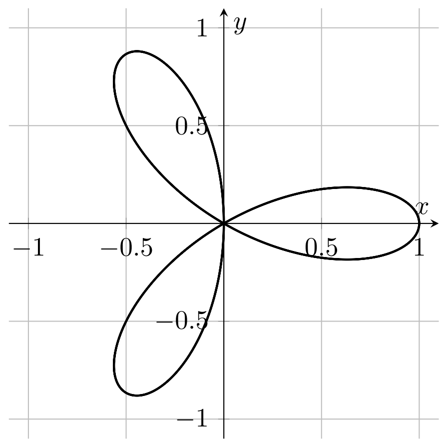
All the previous worksheets are available in seewoo5.github.io/teaching/2026Spring.
Kewords: Polar coordinates
Tips: Use Desmos or Wolfram Alpha to visualize parametric curves.

1. Draw the curve given in polar coordinates $r = \cos 3\theta$. How does it look like?
2. Find the tangent line of the curve $r = 4 + 3\sin\theta$ at $\theta = \pi/6$.
3. Find a polar equation for the line $y = x + 1$.

1. The curve $r = \cos 3\theta$ is a three-petaled rose. Each petal has maximum length 1 and occurs where $\cos 3\theta = 1$, i.e. at

$$3\theta = 2k\pi \Rightarrow \theta = 0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}.$$

The petals appear for values of θ where $\cos 3\theta \geq 0$ (for example around $\theta \in [-\pi/6, \pi/6]$ for the petal centered at 0). Graphically it is symmetric under $\frac{2\pi}{3}$ rotations.



2. For a polar curve $r(\theta)$ we have

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta},$$

where $r' = dr/d\theta$. Here $r = 4 + 3\sin\theta$ so $r' = 3\cos\theta$. At $\theta = \pi/6$ we get

$$r\big|_{\pi/6} = 4 + 3\cdot\tfrac{1}{2} = \tfrac{11}{2}, \qquad r'\big|_{\pi/6} = 3\cdot\tfrac{\sqrt{3}}{2} = \tfrac{3\sqrt{3}}{2}.$$

Thus

$$\frac{dy}{dx}\Big|_{\pi/6} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}\Big|_{\pi/6} = \frac{\frac{3\sqrt{3}}{2}\cdot\frac{1}{2} + \frac{11}{2}\cdot\frac{\sqrt{3}}{2}}{\frac{3\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{11}{2}\cdot\frac{1}{2}} = \frac{\frac{14\sqrt{3}}{4}}{\frac{9-11}{4}} = \frac{7\sqrt{3}/2}{-1/2} = -7\sqrt{3}.$$

The point has Cartesian coordinates

$$x = r\cos\theta = \tfrac{11}{2}\cdot\tfrac{\sqrt{3}}{2} = \tfrac{11\sqrt{3}}{4}, \qquad y = r\sin\theta = \tfrac{11}{2}\cdot\tfrac{1}{2} = \tfrac{11}{4}.$$

Therefore the tangent line is

$$y - \tfrac{11}{4} = -7\sqrt{3}\Big(x - \tfrac{11\sqrt{3}}{4}\Big).$$

3. Using $x = r\cos\theta$, $y = r\sin\theta$, the line $y = x + 1$ becomes

$$r\sin\theta = r\cos\theta + 1 \quad \Rightarrow \quad r(\sin\theta - \cos\theta) = 1.$$

Hence one polar equation is

$$r = \frac{1}{\sin\theta - \cos\theta} = \frac{1}{\sqrt{2}\,\sin\big(\theta - \frac{\pi}{4}\big)}$$

where $\theta \neq \frac{\pi}{4} + k\pi$, for any integer k.