All the previous worksheets are available in seewoo5.github.io/teaching/2026Spring.
Kewords: Polar coordinates
Tips: Use Desmos or Wolfram Alpha to visualize parametric curves.

1. Find a parametrization of a circle given by

$$(x - 2)^2 + y^2 = 4$$

in polar coordinates. Using the parametrization, find the circumference of the circle.

2. Consider the following curves given in polar coordinates:

$$r = 1 + \cos\theta, \quad r = 2 - \cos\theta$$

Find the area of the region that lies inside one curve but outside the other.

3. Find the area of the region bounded by the following two curves given in polar coordinates:
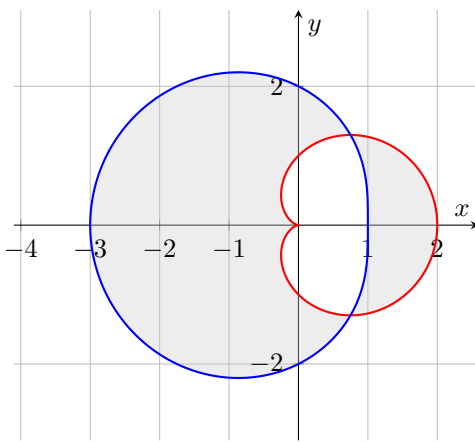
$$r^2 = 2\sin 2\theta, \quad r = 1.$$

1. The circle can be parametrized as $r = 4\cos\theta$, with $-\pi/2 \le \theta \le \pi/2$. Then $dr/d\theta = -4\sin\theta$, so the circumference is

$$\int_{-\pi/2}^{\pi/2} \sqrt{(-4\sin\theta)^2 + (4\cos\theta)^2}\, d\theta = 4\pi.$$

This matches the formula for circumference $2\pi r$ with $r = 2$.

2. The curves intersect when $1 + \cos\theta = 2 - \cos\theta$, so $\cos\theta = \frac{1}{2}$ and $\theta = \pm\frac{\pi}{3}$. For $-\frac{\pi}{3} < \theta < \frac{\pi}{3}$, we have $1 + \cos\theta > 2 - \cos\theta$, and vice versa outside this interval. Thus the area is

$$A = \frac{1}{2} \int_{-\pi/3}^{\pi/3} \left((1 + \cos\theta)^2 - (2 - \cos\theta)^2\right) d\theta + \frac{1}{2}\int_{\pi/3}^{5\pi/3} \left((2 - \cos\theta)^2 - (1 + \cos\theta)^2\right) d\theta = 6\sqrt{3} + \pi$$

3. Intersections of $r^2 = 2\sin 2\theta$ with $r = 1$ satisfy $2\sin 2\theta = 1$, so $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$. On $\frac{\pi}{12} \le \theta \le \frac{5\pi}{12}$ we have $2\sin 2\theta \ge 1$, so the half of the area is

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/12} 2\sin 2\theta\, d\theta + \frac{1}{2}\int_{\pi/12}^{5\pi/12} 1\, d\theta + \frac{1}{2}\int_{5\pi/12}^{\pi/2} 2\sin 2\theta\, d\theta = 1 - \frac{\sqrt{3}}{2} + \frac{\pi}{6}$$

and

$$A = 2 - \sqrt{3} + \frac{\pi}{3}.$$