All the previous worksheets are available in seewoo5.github.io/teaching/2026Spring.
Kewords: 3D coordinates, vectors, dot product, surfaces

1. Find the center and the radius of the sphere given by the equation
$$x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0.$$
2. Let $A(2, 2, 2)$, $B(4, 3, 3)$, and $C(3, 4, 1)$ be points in $\mathbb{R}^3$. Find the cosine of the three angles of the triangle $\triangle ABC$.
3. Find the unit vectors that are orthogonal to the tangent line to the graph of the function $y = x^3$ at the point $(1, 1)$.

1. Complete the square:

$$(x^2+8x)+(y^2-6y)+(z^2+2z)+17=0$$
$$(x+4)^2-16+(y-3)^2-9+(z+1)^2-1+17=0$$
$$(x+4)^2+(y-3)^2+(z+1)^2=9.$$

So the center is $(-4,3,-1)$ and the radius is 3.

2. Use the dot product formula $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}$.

At A: $\overrightarrow{AB}=(2,1,1)$ and $\overrightarrow{AC}=(1,2,-1)$.

$$\overrightarrow{AB}\cdot\overrightarrow{AC}=2\cdot1+1\cdot2+1\cdot(-1)=3,\quad \|\overrightarrow{AB}\|=\|\overrightarrow{AC}\|=\sqrt{6},$$

so

$$\cos\angle A=\frac{3}{6}=\frac{1}{2}.$$

At B: $\overrightarrow{BA}=(-2,-1,-1)$ and $\overrightarrow{BC}=(-1,1,-2)$.

$$\overrightarrow{BA}\cdot\overrightarrow{BC}=(-2)(-1)+(-1)(1)+(-1)(-2)=3,\quad \|\overrightarrow{BA}\|=\|\overrightarrow{BC}\|=\sqrt{6},$$

so

$$\cos\angle B=\frac{1}{2}.$$

At C: $\overrightarrow{CA}=(-1,-2,1)$ and $\overrightarrow{CB}=(1,-1,2)$.

$$\overrightarrow{CA}\cdot\overrightarrow{CB}=(-1)(1)+(-2)(-1)+1\cdot2=3,\quad \|\overrightarrow{CA}\|=\|\overrightarrow{CB}\|=\sqrt{6},$$

so

$$\cos\angle C=\frac{1}{2}.$$

3. The tangent slope is $y'=3x^2$, so at $x=1$ the slope is 3. A direction vector for the tangent line is $(1,3)$. A perpendicular direction vector is $(3,-1)$ (since $(1,3)\cdot(3,-1)=0$). The unit vectors orthogonal to the tangent line are

$$\pm\frac{1}{\sqrt{10}}(3,-1).$$