

All the previous worksheets are available in seewoo5.github.io/teaching/2026Spring.

Keywords: projection, cross product

1. Find the vector projections of \mathbf{b} onto \mathbf{a} .

(a) $\mathbf{a} = \langle 2, 3 \rangle$, $\mathbf{b} = \langle -1, -4 \rangle$

(b) $\mathbf{a} = \langle 1, 2, 2 \rangle$, $\mathbf{b} = \langle 3, 4, -1 \rangle$

2. If $\mathbf{a} = \langle 3, 0, -1 \rangle$, find a vector \mathbf{b} such that $\text{comp}_{\mathbf{a}} \mathbf{b} = 2$.

3. (a) Find all vectors \mathbf{v} such that

$$\langle 1, 2, 1 \rangle \times \mathbf{v} = \langle 3, 1, -5 \rangle$$

- (b) Explain why there's no vector \mathbf{v} such that

$$\langle 1, 2, 1 \rangle \times \mathbf{v} = \langle 3, 1, 5 \rangle$$

1. Vector projections of \mathbf{b} onto \mathbf{a} .

$$(a) \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a} = \frac{-14}{13} \langle 2, 3 \rangle = \left\langle -\frac{28}{13}, -\frac{42}{13} \right\rangle$$

$$(b) \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a} = \frac{9}{9} \cdot \langle 1, 2, 2 \rangle = \langle 1, 2, 2 \rangle$$

2. For $\mathbf{a} = \langle 3, 0, -1 \rangle$, the condition $\text{comp}_{\mathbf{a}} \mathbf{b} = 2$ means $\mathbf{a} \cdot \mathbf{b} = 2|\mathbf{a}| = 2\sqrt{10}$. In (x, y, z) -coordinates this plane is

$$3x - z = 2\sqrt{10}$$

and any point on that plane works. One example is the parallel choice $\frac{2}{\sqrt{10}} \langle 3, 0, -1 \rangle = \left\langle \frac{6}{\sqrt{10}}, 0, -\frac{2}{\sqrt{10}} \right\rangle$.

3. Solve $\langle 1, 2, 1 \rangle \times \mathbf{v} = \langle 3, 1, -5 \rangle$ with $\mathbf{v} = \langle x, y, z \rangle$:

$$(2z - y, x - z, y - 2x) = (3, 1, -5)$$

giving a parametrized line

$$x = 1 + t, \quad y = -3 + 2t, \quad z = t$$

for any t . No \mathbf{v} satisfies $\langle 1, 2, 1 \rangle \times \mathbf{v} = \langle 3, 1, 5 \rangle$ because the right-hand side is not orthogonal to $\langle 1, 2, 1 \rangle$ (dot product = $6 \neq 0$).