All the previous worksheets are available in seewoo5.github.io/teaching/2026Spring.
Keywords: cross product, triple product

1. (a) Show that the four points $P(1,0,2), Q(3,3,3), R(7,5,8), S(5,2,7)$ form a parallelogram.
   (b) Find the area of the parallelogram formed by P, Q, R, S.

2. Check if the vectors
$$\mathbf{u} = \langle 1, 5, -2 \rangle, \quad \mathbf{v} = \langle 3, -1, 0 \rangle, \quad \mathbf{w} = \langle 5, 9, -4 \rangle$$
are coplanar or not.

3. If $\mathbf{a} \cdot \mathbf{b} = \sqrt{3}$ and $\mathbf{a} \times \mathbf{b} = \langle 1, 2, 2 \rangle$, find the angle between $\mathbf{a}$ and $\mathbf{b}$.

4. If $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, show that $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a}$.

1. $\overrightarrow{PQ} = \langle 2, 3, 1\rangle$ and $\overrightarrow{SR} = \langle 2, 3, 1\rangle$, while $\overrightarrow{PS} = \langle 4, 2, 5\rangle$ and $\overrightarrow{QR} = \langle 4, 2, 5\rangle$; opposite sides are parallel and equal, so $PQRS$ is a parallelogram. The area can be computed using the cross product: $\overrightarrow{PQ}\times\overrightarrow{PS} = \langle 13, -6, -8\rangle$, so area $= |\overrightarrow{PQ} \times \overrightarrow{PS}| = \sqrt{269}$.

2. $\mathbf{v} \times \mathbf{w} = \langle 4, 12, 32\rangle$ and $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 4 + 60 - 64 = 0$, so the triple product is zero; the vectors are coplanar.

3. Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. We have $|\mathbf{a}||\mathbf{b}| \cos\theta = \sqrt{3}$ and $|\mathbf{a}||\mathbf{b}| \sin\theta = |\langle 1, 2, 2\rangle| = 3$. Then $\tan\theta = 3/\sqrt{3} = \sqrt{3}$, so $\theta = \pi/3$ (60°).

4. From $\mathbf{a}+\mathbf{b}+\mathbf{c} = 0$ we get $\mathbf{c} = -(\mathbf{a}+\mathbf{b})$. Then $\mathbf{b}\times\mathbf{c} = \mathbf{b}\times[-(\mathbf{a}+\mathbf{b})] = -(\mathbf{b}\times\mathbf{a}) = \mathbf{a}\times\mathbf{b}$. Similarly, $\mathbf{c} \times \mathbf{a} = -(\mathbf{a} + \mathbf{b}) \times \mathbf{a} = \mathbf{a} \times \mathbf{b}$. Hence $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a}$ (note that self-cross products are zero vectors).