All the previous worksheets are available in seewoo5.github.io/teaching/2026Spring.
Keywords: equation of line and plane

1. Find a vector equation, symmetric equation, and parametric equation for the line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$.

2. Find a vector equation of a line that passes through the point $(1, 5, 1)$ and is parallel to the planes $x + 2y + 3z = 1$ and $2x + y + z = 1$.

3. (a) Find the point at which the given lines intersect:

$$\mathbf{r} = \langle 1, 1, 0 \rangle + t\langle 1, -1, 2 \rangle$$
$$\mathbf{r} = \langle 2, 0, 2 \rangle + s\langle -1, 1, 0 \rangle$$

   (b) Find an equation of the plane containing the two lines.

4. Find the distance from the point $(1, -2, 4)$ to the plane $3x + 2y + 6z = 5$.

5. Find the distance between two lines $x = y = z$ and $x + 1 = y/2 = z/3$.

1. A normal vector to $x + 3y + z = 5$ is $\langle 1, 3, 1\rangle$, so a direction vector is $\langle 1, 3, 1\rangle$.

$$\mathbf{r} = \langle 1, 0, 6\rangle + t\langle 1, 3, 1\rangle$$

Parametric form:

$$x = 1 + t, \qquad y = 3t, \qquad z = 6 + t.$$

Symmetric form:

$$\frac{x-1}{1} = \frac{y}{3} = \frac{z-6}{1}.$$

2. The line must be orthogonal to both normals

$$\mathbf{n}_1 = \langle 1, 2, 3\rangle, \qquad \mathbf{n}_2 = \langle 2, 1, 1\rangle,$$

so take direction vector

$$\mathbf{n}_1 \times \mathbf{n}_2 = \langle -1, 5, -3\rangle.$$

Hence

$$\mathbf{r} = \langle 1, 5, 1\rangle + t\langle -1, 5, -3\rangle.$$

3. (a) Write

$$\mathbf{r} = \langle 1 + t, 1 - t, 2t\rangle, \qquad \mathbf{r} = \langle 2 - s, s, 2\rangle.$$

From $2t = 2$, $t = 1$. Then $s = 1 - t = 0$. So intersection point is

$$(2, 0, 2).$$

(b) Direction vectors are

$$\mathbf{v}_1 = \langle 1, -1, 2\rangle, \qquad \mathbf{v}_2 = \langle -1, 1, 0\rangle.$$

So we can take a normal vector to the plane as

$$\mathbf{v}_1 \times \mathbf{v}_2 = \langle -2, -2, 0\rangle.$$

Using point $(2, 0, 2)$:

$$(-2)(x-2) + (-2)(y-0) + 0(z-2) = 0 \quad \Leftrightarrow \quad x + y = 2.$$

4.
$$d = \frac{|3(1) + 2(-2) + 6(4) - 5|}{\sqrt{3^2 + 2^2 + 6^2}} = \frac{18}{7}.$$

5. Write the lines as

$$L_1 : \ \mathbf{r} = \langle 0, 0, 0\rangle + t\langle 1, 1, 1\rangle, \qquad L_2 : \ \mathbf{r} = \langle -1, 0, 0\rangle + s\langle 1, 2, 3\rangle.$$

Let

$$\mathbf{d}_1 = \langle 1, 1, 1\rangle, \qquad \mathbf{d}_2 = \langle 1, 2, 3\rangle, \qquad \mathbf{n} = \mathbf{d}_1 \times \mathbf{d}_2 = \langle 1, 1, 1\rangle \times \langle 1, 2, 3\rangle = \langle 1, -2, 1\rangle.$$

Since $\mathbf{n}$ is perpendicular to both lines, the distance is the length of the projection of a connecting vector onto $\mathbf{n}$. Take

$$\mathbf{w} = \langle -1, 0, 0\rangle - \langle 0, 0, 0\rangle = \langle -1, 0, 0\rangle.$$

Then

$$d = |\mathrm{proj}_{\mathbf{n}}\, \mathbf{w}| = \frac{|\mathbf{w} \cdot \mathbf{n}|}{|\mathbf{n}|} = \frac{|\langle -1, 0, 0\rangle \cdot \langle 1, -2, 1\rangle|}{\sqrt{1^2 + (-2)^2 + 1^2}} = \frac{|-1|}{\sqrt{6}} = \frac{1}{\sqrt{6}}.$$