All the previous worksheets are available in seewoo5.github.io/teaching/2026Spring.
Keywords: vector function

1. At what points does the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t\rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

2. Find a vector function that represents the curve of intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $y + z = 1$. Sketch the curve.

3. Find a parametric equation for the tangent line to the curve

$$\mathbf{r}(t) = \langle \ln(t+1), t\cos 2t, e^t\rangle$$

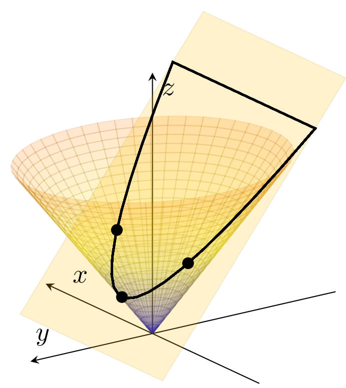
at $t = 0$.

4. If $\mathbf{u}(t) = \mathbf{r}(t) \cdot (\mathbf{r}'(t) \times \mathbf{r}''(t))$, show that

$$\mathbf{u}'(t) = \mathbf{r}(t) \cdot (\mathbf{r}'(t) \times \mathbf{r}'''(t))$$

1. On the sphere,
$$x^2 + y^2 + z^2 = \cos^2 t + \sin^2 t + t^2 = 1 + t^2 = 5,$$
so $t^2 = 4$, hence $t = \pm 2$. Hence the intersection points are
$$(\cos 2, \sin 2, 2), \qquad (\cos 2, -\sin 2, -2).$$

2. From the plane, $z = 1 - y$. From the cone, $z^2 = x^2 + y^2$, so
$$(1-y)^2 = x^2 + y^2 \;\Rightarrow\; x^2 = 1 - 2y \;\Rightarrow\; y = \frac{1 - x^2}{2}.$$
Let $x = t$. Then
$$y = \frac{1-t^2}{2}, \qquad z = 1 - y = \frac{1+t^2}{2},$$
and the corresponding vector function is
$$\mathbf{r}(t) = \left\langle t, \frac{1-t^2}{2}, \frac{1+t^2}{2} \right\rangle, \quad t \in \mathbb{R}.$$
Note that the curve of intersection is a parabola.



3.
$$\mathbf{r}(0) = \langle 0, 0, 1\rangle, \qquad \mathbf{r}'(t) = \left\langle \frac{1}{t+1}, \cos(2t) - 2t\sin(2t), e^t \right\rangle,$$
so
$$\mathbf{r}'(0) = \langle 1, 1, 1\rangle.$$
Thus the equation of the tangent line is given by
$$\mathbf{v}(s) = \langle 0, 0, 1\rangle + s\langle 1, 1, 1\rangle.$$

4. Differentiate using product rule:
$$\mathbf{u}'(t) = \mathbf{r}' \cdot (\mathbf{r}' \times \mathbf{r}'') + \mathbf{r} \cdot (\mathbf{r}'' \times \mathbf{r}'') + \mathbf{r} \cdot (\mathbf{r}' \times \mathbf{r}''').$$
The first term is 0 since $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$, and the second is 0 since $\mathbf{r}'' \times \mathbf{r}'' = \mathbf{0}$. Hence
$$\mathbf{u}'(t) = \mathbf{r}(t) \cdot (\mathbf{r}'(t) \times \mathbf{r}'''(t)).$$