All the previous worksheets are available in seewoo5.github.io/teaching/2026Spring.
Keywords: Partial derivatives

1. Compute f_x and f_y for the following functions:
   (a) $f(x,y) = \ln(x+y+1)$
   (b) $f(x,y) = \sqrt{1-x^2-y^2}$
2. For $f(x,y) = e^{xy^2}$, compute $f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx}$. Check whether $f_{xy} = f_{yx}$.
3. Use implicit differentiation to compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
   (a) $xy + yz + zx = 1$
   (b) $e^z = xyz$
4. For
$$f(x,y) = y\tan^2(x^2) + \frac{x+y}{(x^2+y^2)^{3/2}} e^{\sin(x\sqrt{y})}$$
   find $f_x(1,0)$ and $f_y(0,1)$.

1. (a) For $f(x,y) = \ln(x+y+1)$:

$$f_x = \frac{1}{x+y+1}, \qquad f_y = \frac{1}{x+y+1}.$$

(b) For $f(x,y) = \sqrt{1-x^2-y^2} = (1-x^2-y^2)^{1/2}$:

$$f_x = -\frac{x}{\sqrt{1-x^2-y^2}}, \qquad f_y = -\frac{y}{\sqrt{1-x^2-y^2}}.$$

2. For $f(x,y) = e^{xy^2}$:

$$f_x = y^2 e^{xy^2}, \qquad f_y = 2xye^{xy^2}.$$

$$f_{xx} = y^4 e^{xy^2}, \qquad f_{yy} = (2x + 4x^2y^2)e^{xy^2}.$$

$$f_{xy} = (2y + 2xy^3)e^{xy^2}, \qquad f_{yx} = (2y + 2xy^3)e^{xy^2}.$$

Hence $f_{xy} = f_{yx}$.

3. Let $z = z(x,y)$.

(a) From $xy + yz + zx = 1$:

$$y + y\frac{\partial z}{\partial x} + z + x\frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{y+z}{x+y}.$$

$$x + z + y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial y} = 0 \Rightarrow \frac{\partial z}{\partial y} = -\frac{x+z}{x+y}.$$

(b) From $e^z = xyz$:

$$e^z \frac{\partial z}{\partial x} = yz + xy\frac{\partial z}{\partial x} \Rightarrow \frac{\partial z}{\partial x} = \frac{yz}{e^z - xy}.$$

$$e^z \frac{\partial z}{\partial y} = xz + xy\frac{\partial z}{\partial y} \Rightarrow \frac{\partial z}{\partial y} = \frac{xz}{e^z - xy}.$$

4. For

$$f(x,y) = y\tan^2(x^2) + \frac{x+y}{(x^2+y^2)^{3/2}} e^{\sin(x\sqrt{y})}$$

use the definition of partial derivatives.

First, along $y = 0$:

$$g(x) := f(x,0) = 0 + \frac{x}{(x^2)^{3/2}} e^{\sin(0)} = \frac{x}{|x|^3}.$$

Near $x = 1$, $x > 0$, so $g(x) = \frac{1}{x^2}$. Hence $f_x(1,0) = g'(1) = -2$.

Next, along $x = 0$:

$$h(y) := f(0,y) = y\tan^2(0) + \frac{y}{(y^2)^{3/2}} e^{\sin(0)} = \frac{y}{|y|^3}.$$

Near $y = 1$, $y > 0$, so $h(y) = \frac{1}{y^2}$. Therefore $f_y(0,1) = h'(1) = -2$.